

SUPPLEMENTAL MATERIAL FOR: Fine-scale Droplet Clustering in Atmospheric Clouds: 3D Radial Distribution Function from Airborne Digital Holography

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(Dated: September 24, 2018)

ADDITIONAL DETAILS ABOUT DATA AND INSTRUMENT

Summary Flight Data

Figure S1 shows the time histories of droplet number N , mean diameter, and droplet size distribution for the four flight segments analyzed in the manuscript.

Table S1 shows the flight interval-averaged standard deviations of drop diameter D , Stokes number St , and settling parameter Sv . The range of St is relevant because as it increases, the inertial clustering signature tends to saturate at progressively larger values of r [1]. In other words, the monotonic increase of $g(r)$ with decreasing r reaches a plateau at a critical length scale r_c , and r_c is observed to increase with the width of the particle Stokes number distribution. It is likely that similar effects will hold true for the settling parameter Sv .

TABLE S1. Flight interval-averaged estimates and standard deviations for mean drop diameter (D), Stokes number (St), and settling parameter (Sv). Note that in some cases the standard deviations are comparable to or larger than their corresponding mean values.

Interval Ref	$\langle D \rangle$ (μm)	σ_D (μm)	$\langle St \rangle$	σ_{St}	$\langle Sv \rangle$	σ_{Sv}
A	15.0	5.4	5.5×10^{-3}	7×10^{-3}	0.61	0.76
B	15.0	3.1	6.1×10^{-3}	3×10^{-3}	0.59	0.29
C	16.0	4.5	1.0×10^{-2}	7×10^{-3}	0.54	0.38
D	20.2	4.7	1.3×10^{-2}	7×10^{-3}	0.99	0.55

3D vs. 1D Sampling Uncertainty Volume Parameters

Table S2 gives the details used in the four example sample volumes used for evaluating sampling uncertainty in the estimation of the radial distribution function.

TABLE S2. Properties of the 4 theoretical sample volumes created to explore individual hologram sampling variability and its dependence on measurement volume shape and observed drop concentration

Volume Label	Dimensions of Sample (cm)	Number of Cloud Droplets
3D Volume, 70/cc	0.6 x 0.6 x 10	250
3D Volume, 300/cc	0.6 x 0.6 x 10	1080
1D Volume, 70/cc	0.036 x 0.036 x 2778	250
1D Volume, 300/cc	0.036 x 0.036 x 2778	1080

Instrumental uniformity

Uniformity of particle-detection probability over the instrument sample volume is an important assumption within this work. The first step to that end was to select a subset of the full HOLODEC sample volume that showed uniform particle counts. The result was the 3.6 cm³ volume used in this study. The full sample volume has significant deviations from uniform sensitivity near the sample edges, due to the hologram reconstruction and instrument illumination. Even within the remaining 3.6 cm³ volume, however, there are small fluctuations in instrumental sensitivity. The natural question follows: could the small detected signal result from these small instrumental sensitivity fluctuations?

In order to explicitly test for this, a new ‘enhanced’ numerical simulation was developed and tested on Flight Interval A. This new numerical simulation was constructed very similarly to the method used to generate the gray envelopes in Figure 2 of the manuscript and already described in the main text, but instead of placing each simulated cloud droplet perfectly randomly within the 3.6 cm³ volume the cloud droplets were placed by randomly drawing from one of the 459189 locations actually measured during the entire flight interval. To mimic the data as closely as possible, no two particles in the same hologram were allowed to have the same position — but other holograms were allowed to draw from the entire library of 459189 positions.

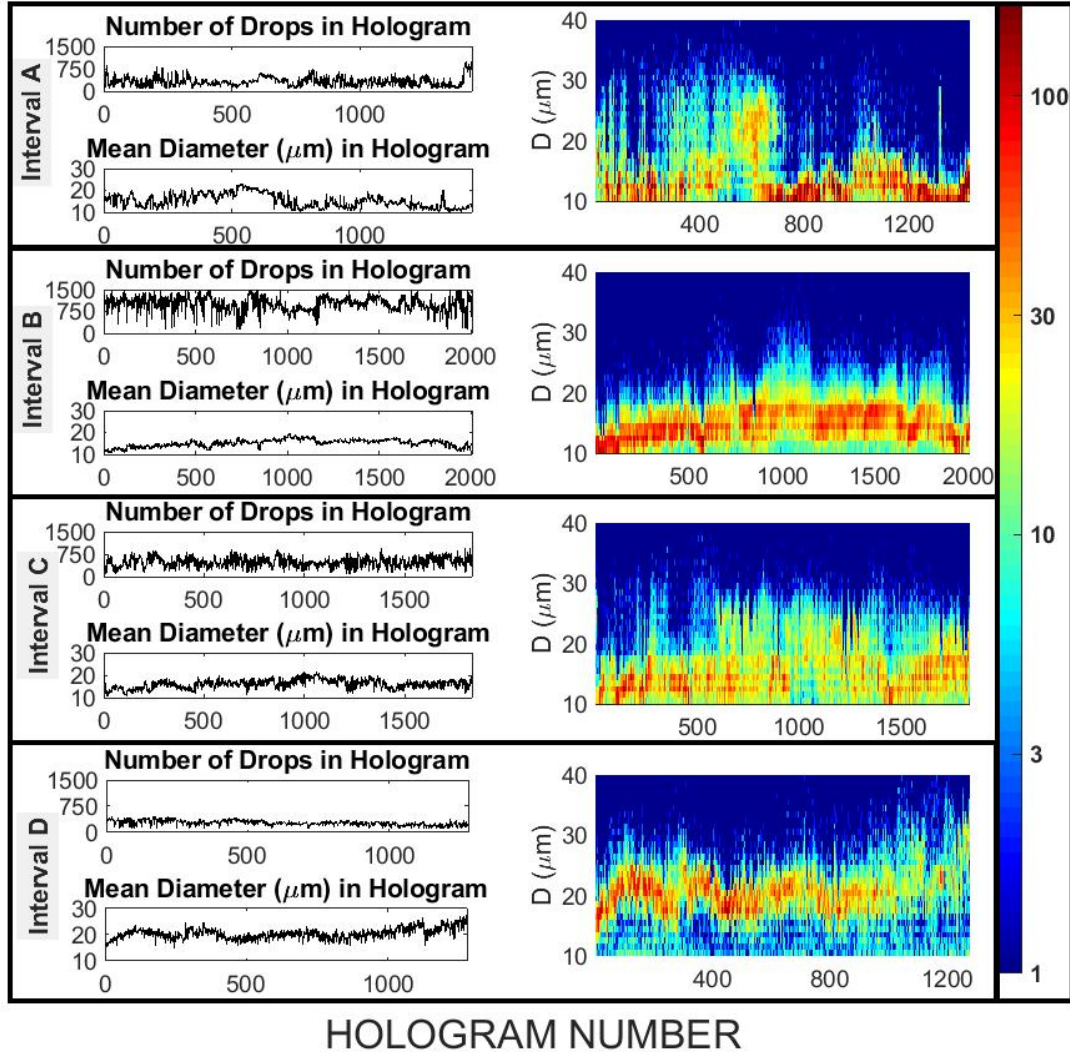


FIG. S1. Each 3-panel box corresponds to summary data from a flight interval. In each box the upper-left plot shows the number of particles in each hologram, the lower-left plot shows the mean droplet diameter for each hologram, and the right panel shows the hologram-by-hologram size distribution with color indicating the number of drops at a given size seen in each hologram (refer to colorbar at far right for scale). All panels have hologram number on the x -axis.

As in the main text, 100 different simulated flights were generated with this method. This simulation is explicitly designed to allow for instrument non-uniformities to appear in the numerical results. Figure S2 summarizes the results from this simulation.

Some changes possibly attributable to instrument non-uniformities are apparent — the red envelope tends to come closer to the observed black curve than gray envelope, suggesting that some of our measured signal may be due to instrumental imperfections. Nevertheless, the black measured rdf curve still lies outside of the entire

red envelope for most scales between 1-5 mm. The observational data still reveals more clustering than could be generated from an unclustered simulation even in the presence of these sources of excess variability.

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[1] E.-W. Saw, J. Salazar, L. Collins, and R. Shaw, *New Journal of Physics* **14** (2012), 10.1088/1367-2630/14/10/105030.

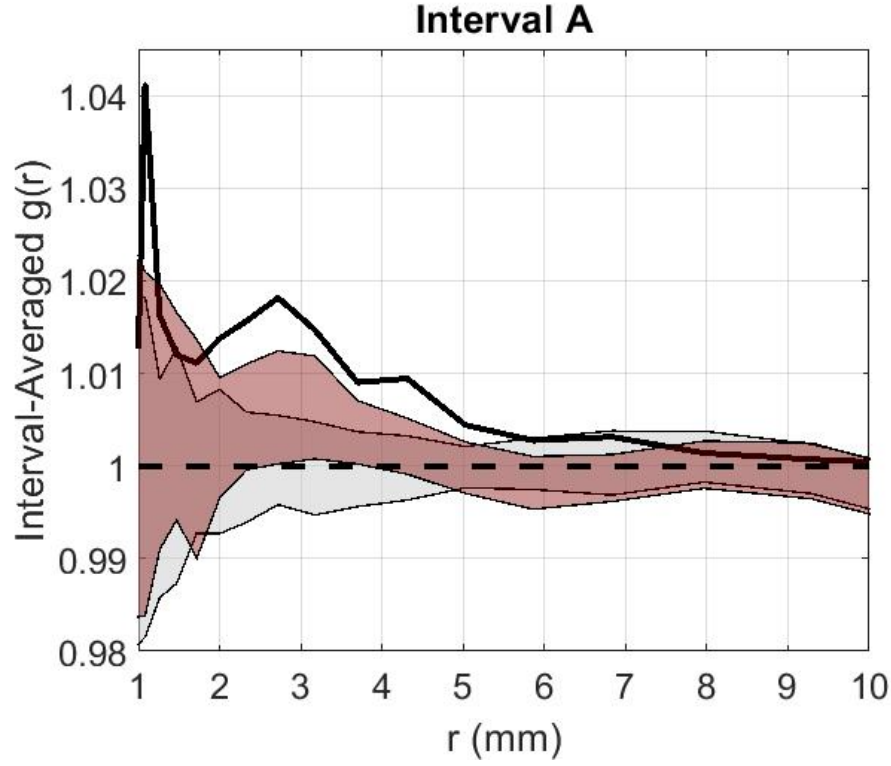


FIG. S2. The results from a test to determine whether the small degree of clustering detected could be due to detection non-uniformity in the instrument. The black line and gray envelope are the same as the upper-left panel of manuscript figure 2, and the $g(r) = 1$ line is marked with a dotted black line. The red region highlights the range of observed flight-averaged rdbs due to random resampling from the observed droplet positions, while still retaining the observed hologram-to-hologram variability of cloud drop number. Although the red envelope approaches the observed black line, the observed rdf still remains outside of the entire ensemble of simulations for most of the range 1-5 mm.